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A NOVEL MULTI-OBJECTIVE MATHEMATICAL MODEL FOR PLANNING AN AUXILIARY PUBLIC TRANSPORTATION NETWORK: A COVERING-ROUTING APPROACH

Abstract. *This paper presents a new application of covering-routing problem for planning of an Auxiliary Public Transportation Network (APTN) to assist conventional network through satisfying additional demand during the time periods of peak traffic. The objective is to plan the APTN so that additional demand coverage is maximized, while the APTN costs are minimized, simultaneously. The main difference of the present work with the other applications of the covering-routing problem is that each bus station can be origin or destination of each passenger. In addition, this research demonstrates the applicability and usefulness of the developed model on a case study on the APTN planning for an urban area in Tehran metropolis, Iran, following different scenarios. It applies ϵ -constraint method to solve the proposed multi-objective model and to present Pareto frontier solutions. This work provides practitioners, specifically planning teams, with a new approach to assist with the APTN planning and improve their logistics decisions.*

Keywords: *Auxiliary public transportation network, Covering-routing problem, Multi-objective optimization.*

JEL Classification: C44, R53, Z18

1. Introduction

In an urban transportation network, there is major travel demand in demand centres (bus stations) during time periods of peak traffic. Hence, this paper

attempts to plan an Auxiliary Public Transportation Network (APT_N) in order to help conventional public network with coverage of additional travel demands as much as possible. Since development of the APT_N and related maintenance operations create huge costs for the urban transportation network, in order to design the APT_N efficiently, this work considers the aforementioned costs and develops a mathematical model to maximize the coverage of additional travel demand and minimize the APT_N costs, simultaneously. However, it is unfortunately possible that residual passengers are covered by neither the APT_N nor the conventional transportation network, and they may use other modes to travel.

Covering-routing problem (CRP) is defined through vehicle routing problem (VRP) and covering problem (CP). VRP refers to a general class of combinatorial optimization problems in which customers are served by a number of vehicles (Mirabiet al. 2010). Suppose a transportation company wants to serve a number of customers. A fleet of vehicles with limited capacity is utilized to transport the products from positions called “depot” to customers. In many VRPs, each vehicle finally returns to the starting depot, while in the others, vehicles do not return to any depot (open VRP) (Erbao and Mingyong 2010). In general, the problem output is the design of a set of routes for a vehicle fleet starting and ending at a depot, in order to serve customers with known demands and minimize the total delivery distance or spent time in serving all customers.

CP is one of the most popular models among facility location concept (Farahani et al., 2012). For the first time, Hakimi (1995) developed a CP to determine the minimum number of police needed to cover nodes on a highways network. In a specific class of CPs called “set covering problems”, demand of a customer should be satisfied by at least one facility within a given critical distance called coverage distance or coverage radius. While in the others, allocated resources are not sufficient to cover all of demand nodes and therefore maximal covering problem maximizes the amount of demand covered within the acceptable service distance by locating a given fixed number of new facilities (Farahani et al., 2012).

In CRPs, the size of facilities like paths and trees is large in proportion to their cover set, and therefore they cannot be assumed as points (Farahani et al., 2012). The main difference between CRP and VRP is that, in CRP there is not necessity to cover all of demands. In CRPs, a path covers a site, if it passes within a distance S (the service distance) of the site. It is notable to mention that in some CRPs there is an existing network whereas in others the network should be constructed (Boffey and Narula, 1998).

The contributions of this work to the literature are two-fold. First, in this paper a new application of covering-routing problem is developed for planning an urban APT_N. Secondly, the presented model incorporates the interaction between passenger’s pick-up and delivery. It means that passengers could get into a bus and land in each bus station. In fact, a delivery of node i consists of passengers getting

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into a bus in previous bus stations while their destination is the i^{th} bus station. Moreover, the present paper solves and validates the proposed model for a real urban APTN in Tehran metropolis based on different scenarios, defined by ε -constraint method, and then compares the computational results together.

The rest of this paper is structured as follows: Section 2 presents the literature review. In Section 3, the problem definition, assumptions and mathematical model are described in details. Solution approach is discussed in Section 4. The case study is elaborated in Section 5. The validity of the model is approved in Section 6. Computational results are analysed in Section 7. Finally, the conclusions and suggestions for future studies are provided

2. Literature review

CRP is defined through CP and VRP. Therefore, the review of these three types of problem is presented in this section.

2.1. Vehicle Routing Problem (VRP)

Clarke and Wright (1964) introduced VRP for the first time, although Golden et al. (1977) presented the first paper including the phrase “vehicle routing” in its title. Later, the other versions of VRP according to microcomputer capability and availability were introduced (Clarke and Wright, 1964). Ghoseiri and Ghannadpour (2010) presented a new model and solution approach for multi-objective VRP with time windows. They considered simultaneous minimization of number of vehicles and total travelling distance as the objective functions. For the first time, Mendoza et al. (2010) introduced a multi-compartment VRP with stochastic demands. In the multi-compartment VRP, products are incompatible and must be transported in independent compartments of vehicle. Sáez et al. (2008) presented a dynamic multi-vehicle pick-up and delivery problem formulated under a hybrid predictive adaptive control scheme. The demand prediction in this problem is based on a systematic fuzzy clustering methodology. Erbao and Mingyong (2010) presented an open VRP with fuzzy demands which involves recourse action. Angelelli and Speranza (2002) introduced an extension of periodic VRP for a collection problem. In their proposed model, vehicles can renew their capacity at some intermediate facilities. Tricoire et al. (2013) introduced a multi-period, multi-depot un-capacitated VRP with specific constraints to plan customer service operations and schedules of the technicians over the horizon time called multiple field service routing problem (MPFSRP). They proposed an exact branch and price solution method and several heuristic approaches for small-sized and larger instances respectively. Khebbache-Hadji et al. (2013) developed a two-dimensional loading cutting and packing VRP with time windows (2L-CVRPTW). This problem consists in determining vehicle trips to deliver rectangular objects to a set of customers with known time windows. They designed six heuristic

approaches to quickly compute good solutions, and after that to improve the heuristic solutions, a Memetic algorithm was applied.

2.2. Covering Problem (CP)

For the first time, Hakimi(1965) developed a CP to determine minimum number of polices needed to cover the nodes on a network of high ways. Yin and Mu (2012)developed a new maximal covering location problem (MCLP) for emergency vehicles in which several possible capacity levels for the facility at each potential site are available; called Modular Capacitated MCLP. This model has been developed in two situations: the MCLP facility-constraint and the MCLP non-facility-constraint. Davari et al.(2011)introduced a MCLP in which travel time between any pair of nodes is a fuzzy variable.

2.3. Covering-routing Problem (CRP)

Mesa and Boffey (1996) developed a review of extensive facilities which are in essence 1-dimansional such as paths and trees. Boffey and Narula (1998) introduced path covering problem to find a route through the network such that the route length was minimized and the population coverage was maximized. Tricoire et al. (2012) presented a bi-objective covering tour model with stochastic demand. The two objectives in that paperwere; operating cost for distribution centres plus routing cost for a fleet of vehicles, and expected uncovered demand. The model was based on humanitarian logistics that depends on distance, so that a certain percentage of clients come from their homes to the nearest distribution centres. Lamosa et al. (2011) introduced an interactive approach to solve a multi-objective CRP in which a set of areas should be revisited as soon as possible; called critical areas. Keskin et al. (2012) developed an integrated maximum covering and patrol routing problem in order to determine the patrol routes of state trooper patrols to achieve maximum coverage of highway spots with high frequencies of crashes (called hot spot).

According to the literature review, most of the previous works in the vehicle routing, covering and covering-routing areas focused on dichotomy transportation networks or modelling the structures similar to it. In these models, the vehicles pick up the goods from demand nodes and transport them to one or more specific nodes called depots, or deliver the goods to demand nodes from one or more depots. Also for this reason, the models provided in passenger transportation area, such as the school bus routing problem, are not applicable for the urban public transportation networks (Park and Kim, 2010). Although some of the dichotomy transportation structures are similar to public transportation network models, such as the many-to-many location routing problem, they are not usable for the urban public transportation problems. For example, in many-to-many location routing problem, the goods transported through a hub network (Nagy and Salhi, 2007). Hence, this paper designs an APTN using covering-routing approach to help the conventional transportation network and cover the additional demands during time periods of peak traffic. The main difference between the present

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research and the other covering-routing applications, used for commodities transportation networks, is that passengers could get into a bus and land in each bus station. In fact, a delivery of node i consists of passengers getting into a bus in previous bus stations and their destination is the i^{th} bus station.

3. Problem description and formulation

In the case study that this paper considers, it is elaborated in Section 5, a public transportation company has decided to develop an APTN in an urban area. In this area, there are several existing bus stations where the APTN should be designed over them. The objective is to design an APTN to maximize the additional demands coverage and minimize the APTN costs. The APTN costs consist of investment cost to purchase the buses, maintenance costs and path traversal costs.

To develop the APTN, this paper considers each bus station has a deterministic additional travel demand in the time periods of peak traffic for each destination. In the APTN, each bus starts from the APTN centre (the APTN centre corresponds to a certain bus station), visits and services a number of bus stations and finally goes back to the first origin. Each bus station can be serviced by more than a bus. A vehicle service consists of pick-up and delivery in each bus station of its route. All buses are considered identical and obviously capacitated. A logical constraint is that a bus picks up a passenger, with destination x , only when its route includes this destination. In addition to the above description, the other assumptions of the problem are as follows:

- Buses have same capacity,
- Each bus covers at most one route in the APTN,
- Each bus station can be serviced by more than one bus,
- The start and finish node of all routes is demand node 1 (the APTN centre),
- The budget limitation for the APTN costs is considered,
- There is an upper bound for the number of buses; K .

The notations and mathematical formulation are given in details by the upcoming subsections.

3.1. Sets

There are only two sets in the problem:

- I Set of bus stations,
- K Set of buses.

3.2. Parameters

The parameters, which are predetermined and given in this problem, are listed below:

d_{ij}	Cost of path traversal between demand node i and j , directly ($i \neq j$),
c	Purchase cost of a bus,
C	Initial capacity of a bus,
D_{il}	Additional travel demand from origin i to destination l ($i \neq l$),
T_{ij}	Duration of arc (i,j) ,
m	Bus maintenance costs for the first year,
B	Available budget,
S	Minimum coverage of demand,
M	Sufficient large number,
N	Number of demand nodes save the APTN center.

3.3. Decision variables

The unknown variables are:

x_{ilk}	Number of passengers that travel from origin i to destination l through bus k ($i \neq l$),
z_{ijk}	1; if bus k covered arc (i, j) , and 0; otherwise ($i \neq j$),
s_k	1; if bus k assigned to the APTN, and 0; otherwise,
C_{ik}	Remaining capacity of bus k when enter the node i ,
t_{ik}	Entrance time of bus k to node i ,
z'_{ilk}	A binary integer, equal to 1, if bus k visited node l after node i ($i \neq l$), and 0 otherwise,
U_{ik}	Auxiliary variable for sub-tour elimination constraints for route related to bus k .

3.4. Mathematical model

The problem is mathematically modelled by the following objectives:

$$\max \sum_{i \in I} \sum_{l \in I} \sum_{k \in K} x_{ilk} \quad (1)$$

$$\min \sum_{i \in I} \sum_{j \in I} \sum_{k \in K} d_{ij} z_{ijk} + \sum_{k \in K} (m + c) s_k \quad (2)$$

Objective function (1) maximizes the additional travel demand coverage while objective function (2) minimizes the APTN costs consisting of investment cost to purchase buses, maintenance costs and path traversal costs.

The constraints are as follows:

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$$\sum_{i \in I} z_{ijk} = \sum_{i \in I} z_{jik} \leq 1, \quad j \in I, k \in K \quad (3)$$

Constraint (3) states the balance at each bus station so that each bus k that visits bus station j must leave it. Also, it ensures that each bus could service each bus station at most once.

$$s_k = \sum_{j \in I} z_{1jk}, \quad k \in K \quad (4)$$

$$\sum_{\substack{i \in I \\ i \neq 1}} \sum_{\substack{j \in I \\ j \neq 1}} z_{ijk} \leq M \sum_{j \in I} z_{1jk}, \quad \forall k \in K \quad (5)$$

Constraint (4) defines s_k variables. Constraint (5) guarantees existence of the APTN center in each route.

$$\sum_{k \in K} x_{ilk} \leq D_{il}, \quad i, l \in I \quad (6)$$

Expression (6) addresses the limitation of the additional demand. This constraint expresses that sum of the passengers that travel from origin i to destination l , should be less than the related additional demand.

$$x_{ilk} \leq M z'_{ilk}, \quad i, l \in I \text{ and } i \neq l \neq 1, k \in K \quad (7)$$

Constraint (7) obligates two conditions for travel from origin i to destination l through bus k . The first condition is that nodes i and l should be over the route related to bus k , and the second is that node l should be after node i .

$$\sum_{i \in I} \sum_{l \in I} \sum_{k \in K} x_{ilk} \geq S \quad (8)$$

$$C_{ik} - C_{jk} \geq \sum_{l \in I} x_{ilk} - \sum_{l \in I} x_{lik} - M(1 - z_{ijk}), \quad i, j \in I \text{ and } i \neq 1, k \in K \quad (9)$$

$$C_{ik} - C_{jk} \leq \sum_{l \in I} x_{ilk} - \sum_{l \in I} x_{lik} + M(1 - z_{ijk}), \quad i, j \in I \text{ and } i \neq 1, k \in K \quad (10)$$

$$C - C_{jk} \geq \sum_{l \in I} x_{ilk} - M(1 - z_{ijk}), \quad j \in I \text{ and } i = 1, k \in K \quad (11)$$

$$C - C_{jk} \leq \sum_{l \in I} x_{ilk} + M(1 - z_{ijk}), \quad j \in I \text{ and } i = 1, k \in K \quad (12)$$

$$C_{ik} \leq C, \quad i \in I, k \in K \quad (13)$$

Expression (8) ensures the minimum coverage of demand. Expressions (9-13) are logical constraints that define remaining capacity of bus k when enters bus station. For $i \neq 1$, if arc (i, j) belongs to the route related to bus k , the difference between C_{jk} and C_{ik} is equal to the difference between the number of passengers with destination i and the number of passengers with origin i are served by bus k . It should be noted that variable C_{1k} define the remaining capacity of bus k when enters the APTN center at the end of the route.

$$t_{jk} - t_{ik} \leq T_{ij} + M(1 - z_{ijk}), \quad i, j \in I \text{ and } j \neq 1 \quad (14)$$

$$t_{jk} - t_{ik} \geq T_{ij} - M(1 - z_{ijk}), \quad i, j \in I \text{ and } j \neq 1 \quad (15)$$

$$t_{ik} \leq M \sum_{j \in I} z_{ijk}, \quad i \in I \text{ and } i \neq 1 \quad (16)$$

Constraints (14-16) define an auxiliary variable named t_{ik} . If arc (i, j) belongs to the route related to bus k , the difference between the entrance time of bus k to node j and node i is equal to the duration of arc (i, j) . If bus k does not visit node i , variable t_{ik} would be equal to zero.

$$t_{lk} - t_{ik} \geq M(z'_{ilk} - 1) - M\left(1 - \sum_{j \in I} z_{ljk}\right), \quad i, l \in I \text{ and } i, l \neq 1 \quad (17)$$

$$t_{lk} - t_{ik} \leq Mz'_{ilk} + M\left(1 - \sum_{j \in I} z_{ijk}\right), \quad i, l \in I \text{ and } i, l \neq 1 \quad (18)$$

$$z'_{ilk} \leq \sum_{j \in I} z_{ijk}, \quad i, l \in I \text{ and } k \in K \quad (19)$$

$$z'_{ilk} \leq \sum_{j \in I} z_{ljk}, \quad i, l \in I \text{ and } k \in K \quad (20)$$

$$z'_{ilk} = z'_{l1k} = \sum_{j \in I} z_{ljk}, \quad i, l \in I \text{ and } k \in K \quad (21)$$

Constraints (17-21) define variable z'_{ilk} . This set of constraints guarantee that variable z'_{ilk} is equal 1; if and only if nodes i and l are over the route related to bus k , and node l is after node i .

$$\sum_{i \in I} \sum_{j \in I} \sum_{k \in K} d_{ij} z_{ijk} + \sum_{k \in K} (m + c) s_k \leq B \quad (22)$$

$$U_{ik} - U_{jk} + N z_{ijk} \leq N - 1, \quad i, j \in I, k \in K \quad (23)$$

Constraint (22) guarantees that the APTN costs do not exceed the available budget. Constraint (23) certifies there is no sub-tour for any bus.

$$M(1 - z'_{ijk}) + x_{ijk} D_{il} \geq x_{ilk} D_{ij} + M(z'_{ilk} - 1), \quad i, j, l \in I \quad (24)$$

$$M(z'_{ijk} - 1) + x_{ijk} D_{il} \leq x_{ilk} D_{ij} + M(1 - z'_{ilk}), \quad i, j, l \in I \quad (25)$$

Constraint (24) and (25) imply that the number of passengers for each destination that are served at bus station i through bus k , is proportional to the additional travel demand from origin i to destination l .

$$x_{ijk}, C_{ik}: \text{positive integer variables, } i, j \in I, k \in K \quad (26)$$

$$z'_{ilk}, s_k, z_{ijk} \in \{0, 1\}, \quad i, j, l \in I, k \in K \quad (27)$$

$$U_{ik} \geq 0, \quad i \in I, k \in K \quad (28)$$

Constraints (26-28) show types of the variables.

4. Solution approach

A multi-objective problem (MOP) can be defined as follows:

$$\min F(x) = (f_1(x), f_2(x), f_3(x), \dots, f_k(x))$$

Subject to

$$x \in D,$$

Meaning that x belongs to the solution space D , and the objective space can be expressed as follows:

$$Y = \{\tilde{y} = (y_1, y_2, \dots, y_k) : y_i = f_i(\tilde{x}), \forall \tilde{x} \in D, i = 1, 2, \dots, k\}$$

In general, Pareto and Scalar methods are two main categories for solving MOPs. Pareto methods directly apply the concept of Pareto dominance. In many cases, the Pareto concept is used within an evolutionary framework. Scalar methods consist of weighted linear aggregation, goal programming, ε -constraint etc. Moreover, non-Scalar and non-Pareto algorithms include Vector Evaluated Genetic Algorithm (VEGA), Lexicographic methods, ant colony systems, specific heuristics, etc. (Jozefowicz et al., 2008).

Boffey and Narula (1998) proposed two solution approaches for solving the maximal population shortest path problem (i.e. weighting method with Lagrangian relaxation and K-shortest path approach). Lamosa et al. (2011) developed an interactive approach in which weighted linear aggregation with 2-opt algorithm were used to find the Pareto optimal set. This approach considers the human interferences in some phases of the process to achieve better generation and evaluation of the solutions of the multi-objective problem. Tricoire et al. (2012) applied a branch and cut technique within ε -constraint algorithm. Doerner et al. (2007) suggested three solution approaches for a multi-objective tour planning: P-ACO technique, VEGA and MOGA variant of multi-objective genetic algorithms. P-ACO is a multi-criteria meta-heuristic approach that simultaneously considers the location and routing aspects of the problem. Jozefowicz et al. (2007) implemented a solution approach in which the solution of a multi-objective evolutionary algorithm for covering tour problem was used as an input for a branch-and-cut algorithm. Jozefowicz et al. (2008) presented some suggestions to design multi-objective solution methods for multi-objective VRPs. They believed that using the different methods in cooperation appears to be successful to solve the multi-objective VRPs. Grandinetti et al. (2012) proposed an optimization-based heuristic for solving multi-objective undirected capacitated arc routing problem (MUCARP). The MUCARP Pareto front is determined through applying of the ε -constrain method.

Among the exact methods to find efficient set of multi-objective problems, weighted sum scalarization is the most popular, which solves different single objective sub-problems generated by a linear scalarization of the objectives. Furthermore, ε -constraint method is the best known approach for solving these problems according to Ehrgott and Xavier (2002). Some of the advantages of this approach are as follows:

1. This approach alters the original feasible region and is able to produce non-extreme efficient solutions. As a consequence, with the ε -constraint method we can exploit almost every run to produce a different efficient solution.

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2. The ε -constraint approach produces unsupported efficient solutions in multi-objective integer and mixed integer programming problems.
3. We can control the number of the generated efficient solutions by properly adjusting the number of grid points in each one of the objective function ranges.

Decision maker in our case study (in an urban transportation company) prefers to achieve a set of non-dominated solutions. Hence, the efficient set of the proposed model is determined by applying ε -constraint algorithm whose main idea is to solve a sequence of constrained single-objective problems.

In order to elaborate ε -constraint algorithm, the following preliminary definitions are explained as follows:

Definition 1. *Dominance relation:* Suppose $\tilde{y}$ and $\tilde{y}' \in Y$, $\tilde{y}$ dominates $\tilde{y}'$ ($\tilde{y} < \tilde{y}'$) if and only if $y_i \leq y'_i, i=1, \dots, k$, where at least one inequality is strict.

Definition 2. *Pareto efficiency:* $\tilde{x}$ is a Pareto efficiency solution in D if and only if $\nexists \tilde{x}'$ such that $f(\tilde{x}) < f(\tilde{x}')$.

Definition 3. *Efficient set:* Efficient set A is defined as follows $A = \{\tilde{x}: \tilde{x} \in D\}$ is Pareto efficient in X ,

Definition 4. *Pareto front:* Pareto front F is defined as follows $F = \{f(\tilde{x}): \tilde{x} \in A\}$.

Definition 5. *Ideal point:* The vector $\tilde{Z}^I = (z_1^I, z_2^I)$ such that $z_1^I = \min_{\tilde{z} \in Y} z_1$ and $z_2^I = \min_{\tilde{z} \in Y} z_2$; represents ideal point,

Definition 6. *Nadir point:* The vector $\tilde{Z}^N = (z_1^N, z_2^N)$ such that $z_1^N = \min_{\tilde{z} \in Y} \{z_1: z_2 = z_2^I\}$ and $z_2^N = \min_{\tilde{z} \in Y} \{z_2: z_1 = z_1^I\}$; represent the nadir point.

The main idea of ε -constraint algorithm is to define and solve a set of ε -constraint problems, $P_i(\varepsilon)$. Thus in the bi-objective case, this approach solves two optimization problems:

$P_1(\varepsilon_2)$:

$$\begin{aligned} \min f_1(\tilde{x}) \\ \tilde{x} \in D \\ f_2(\tilde{x}) \leq \varepsilon_2 \end{aligned}$$

$P_2(\varepsilon_1)$:

$$\begin{aligned} \min f_2(\tilde{x}) \\ \tilde{x} \in D \\ f_1(\tilde{x}) \leq \varepsilon_1 \end{aligned}$$

ε -constraint framework for the case of two objectives is expressed in Algorithm section as follows:

Solution Algorithm: Bi-objective ε -constraint method framework:

1. Set $i=1, j=2$ or $i=2, j=1$,
2. Compute ideal and nadir points,

3. Set $O' = \{(z_i^l, z_j^N)\}$ and $\varepsilon_j = z_j^N - \Delta$. ($\Delta = 1$)
4. While $\varepsilon_j \geq z_j^l$, do:
 - (a) Solve $P_i(\varepsilon_j)$ and add the optimal solution value (z_i^*, z_j^*) to O' .
 - (b) Set $\varepsilon_j = z_j^* - \Delta$
5. Remove dominated points from O' if required.

5. Numerical experiment: Actual case study

In order to verify and show effectiveness of the proposed model, we present a case study in an urban area in Tehran metropolis in Iran. A public transportation company is selected to service this area with 16 demand nodes (bus stations). There is straight route between some of these nodes (see Fig. 1). The company can choose one of three alternatives, scenarios, with different type of bus and available amount of budget for the APTN costs. Table 1 shows the parameters of these scenarios. The maximum number of routes can be developed ($|K|$), is assumed 1. Also, minimum demand must be covered (S) is considered 20. Tables 2 to 4 illustrate cost of path traversal, travel additional demands and duration of straight paths between bus stations respectively.



Figure 1. Initial public transportation network

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Table 1. Parameters of the scenarios

	Scenarios		
	#1	#2	#3
Initial capacity of a bus; C (person)	60	140	180
Bus maintenance costs for the first year; m (\$)	3000	5440	6500
Purchase cost of a bus c (\$)	30000	54400	65000
B (\$)	65000	90000	100000

Table 2. Cost of path traversal between demand nodes

$d_{ij}(\text{\$})$	Bus stations															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	4092	3108	-	-	-	-	-	-	-	-	-	-	-	-	-
2	-	0	3564	2132	-	-	-	-	-	-	-	-	-	-	-	-
3	3108	4300	0	3200	-	-	1712	-	-	-	-	-	-	-	-	-
4	-	-	-	0	-	-	3388	-	-	-	3580	-	-	-	-	-
5	-	-	-	-	0	-	-	-	1440	-	-	-	-	-	-	-
6	-	-	-	-	2108	0	-	1568	-	-	-	-	-	-	-	-
7	-	-	1712	-	-	2344	0	-	-	-	-	4708	-	-	-	-
8	-	-	-	-	-	-	-	0	-	2680	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	0	5540	-	-	-	-	-	5792
10	-	-	-	-	-	-	-	-	-	0	-	2208	-	-	-	-
11	-	-	-	-	-	-	-	-	-	-	0	-	2808	-	-	-
12	-	-	-	-	-	-	4708	-	-	-	-	0	-	2504	-	-
13	-	-	-	-	-	-	-	-	-	-	-	4092	0	-	-	-
14	-	-	-	-	-	-	-	-	-	-	-	2504	-	0	-	-
15	-	-	-	-	-	-	-	-	-	-	-	-	-	3484	0	-
16	-	-	-	-	-	-	-	-	-	5792	-	-	-	-	5156	0

Table 3. Additional demand of bus station

D_{il}	Bus stations															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	9	8	7	5	5	10	6	1	14	9	3	7	4	10	4	6
2	0	4	7	4	12	8	0	6	14	3	12	1	6	6	7	13
3	1	5	8	7	11	2	8	3	9	8	13	6	9	1	7	1
4	7	13	14	12	15	13	0	7	6	11	2	7	6	14	1	11
5	13	0	2	1	7	10	4	3	7	11	14	15	13	0	8	4
6	8	7	13	14	8	13	1	12	12	0	8	12	14	5	4	8
7	14	12	7	14	12	3	3	10	12	15	1	1	0	11	14	13
8	15	5	13	10	0	8	6	10	14	1	11	4	11	4	14	6

9	8	10	5	6	2	14	2	1	15	13	3	9	14	1	1	9
10	9	10	9	1	11	12	9	4	10	1	3	6	1	2	7	12
11	4	3	9	14	4	15	10	3	15	15	7	4	2	6	7	2
12	8	11	11	8	8	14	8	5	2	0	6	14	13	14	5	7
13	12	2	7	0	3	12	3	7	9	14	2	13	4	9	2	0
14	2	9	1	2	9	13	10	7	5	0	11	12	1	14	8	2
15	1	6	8	0	1	7	10	11	2	7	8	3	3	2	0	15
16	9	4	13	15	9	1	2	11	10	7	6	14	14	4	14	6

Table 4. Duration of straight paths

T_{ij}	Bus stations															
	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
1	0	205	155	-	-	-	-	-	-	-	-	-	-	-	-	-
2	-	0	178	107	-	-	-	-	-	-	-	-	-	-	-	-
3	155	215	0	160	-	-	86	-	-	-	-	-	-	-	-	-
4	-	-	-	-	-	-	169	-	-	-	179	-	-	-	-	-
5	-	-	-	-	-	-	-	-	72	-	-	-	-	-	-	-
6	-	-	-	-	105	-	-	78	-	-	-	-	-	-	-	-
7	-	-	86	-	-	117	-	-	-	-	-	235	-	-	-	-
8	-	-	-	-	-	-	-	-	-	134	-	-	-	-	-	-
9	-	-	-	-	-	-	-	-	-	277	-	-	-	-	-	290
10	-	-	-	-	-	-	-	-	-	-	-	110	-	-	-	-
11	-	-	-	-	-	-	-	-	-	-	-	-	140	-	-	-
12	-	-	-	-	-	-	235	-	-	-	-	-	-	125	-	-
13	-	-	-	-	-	-	-	-	-	-	-	205	-	-	-	-
14	-	-	-	-	-	-	-	-	-	-	-	125	-	-	-	-
15	-	-	-	-	-	-	-	-	-	-	-	-	-	174	-	-
16	-	-	-	-	-	-	-	-	290	-	-	-	-	-	258	-

6. Validation

In this section, we investigate the effectiveness of the model considering one objective (demand coverage) through sensitivity analysis of main parameters (i.e., d_{ij} and C) for the first scenario. Therefore, the model is solved with single demand coverage objective in LINGO software. The computational results of the sensitivity analysis of parameter C provided in Table 5 and Fig. 2.

When the bus capacity increases, we expect that the demands covered through developed APTN, increase non-strictly. Therefore, Fig. 2 illustrates a logical trend. Table 6 and Fig. 3 shows the sensitivity analysis results for the parameter d_{ij} .

Table 6 shows the amount of demand coverage while the value of parameter d_{ij} is changed ten percent every time. The APTN model solved for each value of cost of path traversal between demand nodes (d_{ij}) with single objective, demand coverage. According to budget limitation, as d_{ij} increases, the demand

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coverage should be decrease non-strictly. Therefore, Fig. 3 demonstrates a non-ascending diagram that is a logical trend.

Table 5. Sensitivity analysis for the parameter C

Number	Capacity	Objective #1 (demand coverage)
1	40	70
2	50	89
3	60	108
4	70	151
5	80	154
6	90	160
7	100	190
8	110	190
9	120	219
10	130	219

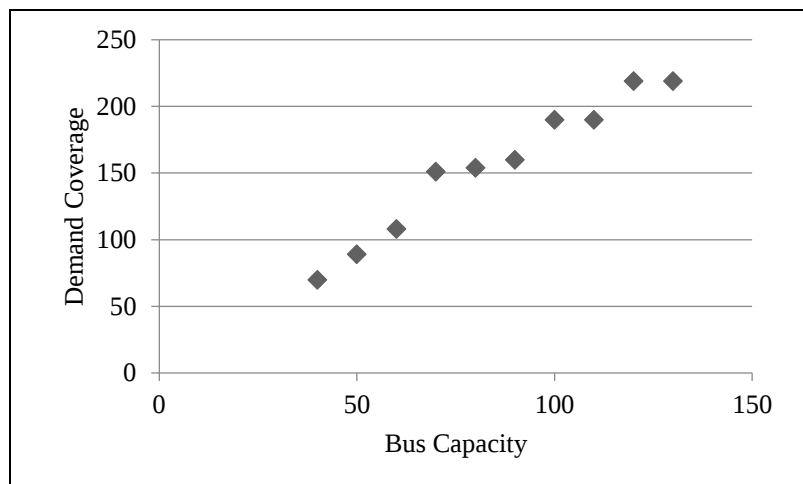


Figure 2. Sensitivity analysis for the parameter C

Table 6. Sensitivity analysis for the parameter d_{ij}

Number	d_{ij} change(%)	Objective 1 (demand coverage)
1	-40	108
2	-30	108
3	-20	108
4	-10	108
5	0	108
6	10	108
7	20	90
8	30	80
9	40	54
10	50	54

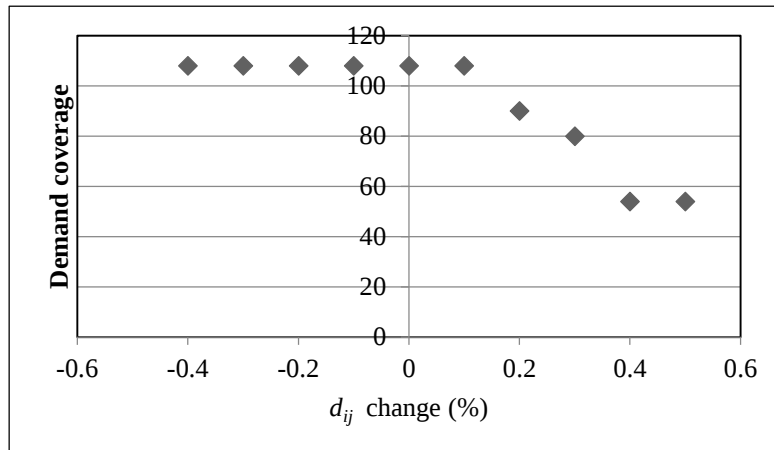


Figure 3. Sensitivity analysis for parameter d_{ij}

7. Computational results

As previously mentioned, the model is solved by the ε -constraint method for each scenario in LINGO software. Table 7 represents the ideal and nadir points of the objectives for Scenario #1. Table 8 shows the value of objectives for each value of ε parameter. The highlighted rows in Table 8, shows the Pareto front of the problem for Scenario #1. Fig. 4, 5 and 6 illustrate the diagrams of the Pareto front for Scenario #1, #2 and #3.

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Table 7. Upper and lower bound (Scenario #1)

Ideal & Nadir points	Objective #1 (demand coverage)	Objective #2 (APTN costs)
Upper bound	$z^L=108$	$z^N=59232$
Lower bound	$z^N=22$	$z^L=43764$

Table 8. Computational results of ε -constraint algorithm (Scenario #1)

Number	ε	Objective #1 (demand coverage)	Objective #2 (APTN cost)
1	-	22	43764
2	23	23	43764
3	24	26	47432
4	27	33	47432
5	34	34	47432
6	35	36	47432
7	37	47	47432
8	48	54	47432
9	55	57	47432
10	58	58	47432
11	59	79	47432
12	80	80	47432
13	81	81	59232
14	82	92	59232
15	93	97	59232
16	98	99	59232
17	100	101	59232
18	102	108	59232

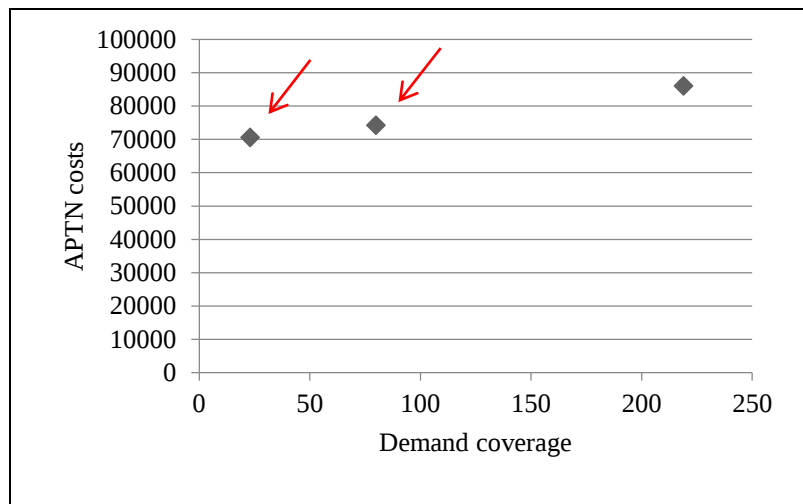


Figure 4. Pareto front for Scenario #1

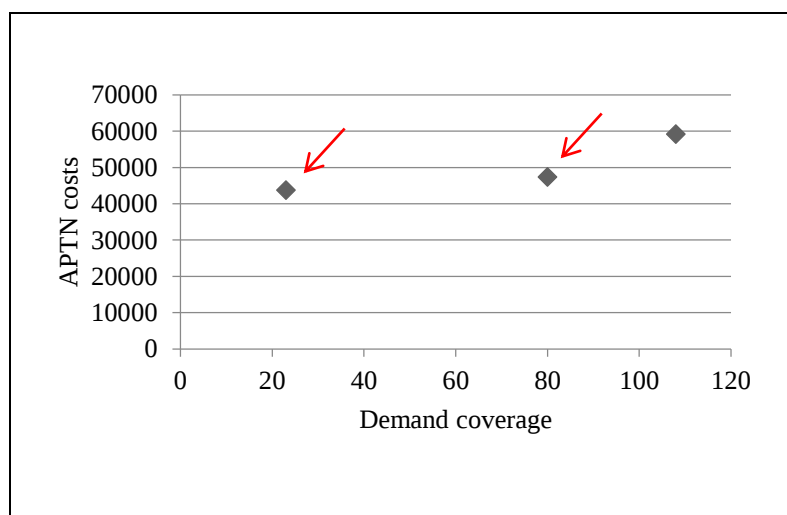


Figure 5. Pareto front for Scenario #2

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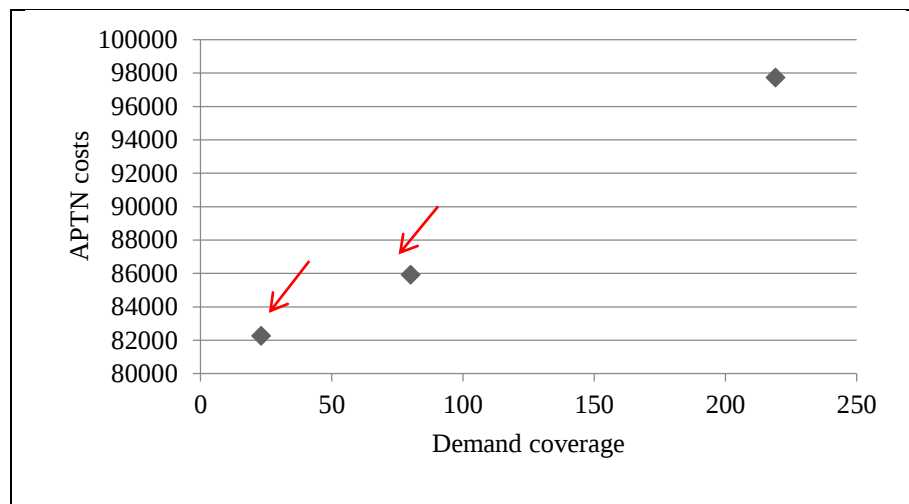


Figure 6. Pareto front for Scenario #3

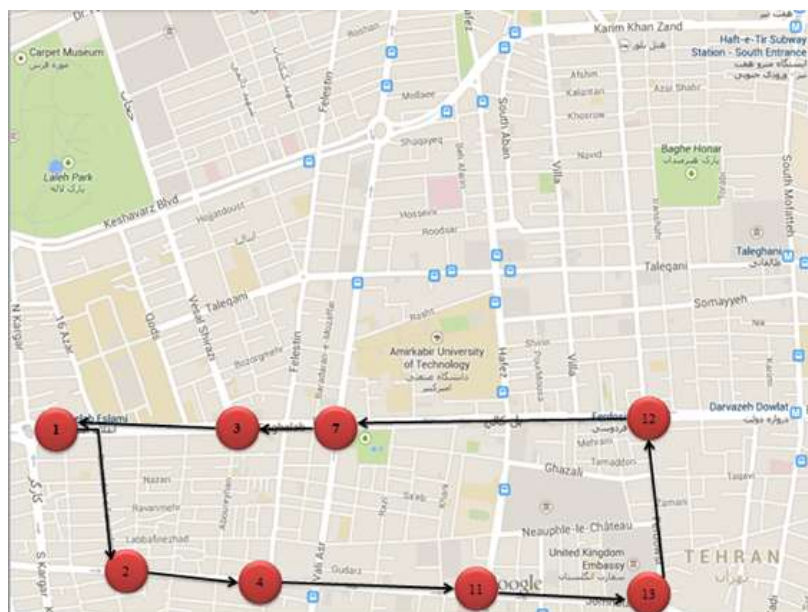


Figure 7. Optimally designed route of the third point of the Pareto front for Scenario #1

As illustrated in Fig. 4-6, two points of the Pareto front in Scenario #1, #2 and #3 have the same value of demand coverage, while the APTN costs are different-Scenario #1 results in lowest APTN costs for these two points. Also, for Scenario #2 and #3, the value of the first objective is equal for the all three points of the Pareto front with lower APTN costs for Scenario #2. Therefore, Scenario #2 is better than third one. Fig.7 illustrates the optimally designed route of the third point of the Pareto front for Scenario #1.

Finally, in order to determine the best solution among the Pareto fronts of the scenarios, we should get the secondary criteria from the related public transportation company. For example, the company may consider the average cost of APTN for each passenger covered through this network as secondary criteria. Thus, according to this criterion, the third point of the Pareto front for Scenario #2 is the best. Fig 8 and 9 shows the optimally designed route of the second and third point of the Pareto front for Scenario #2 and #3 respectively.

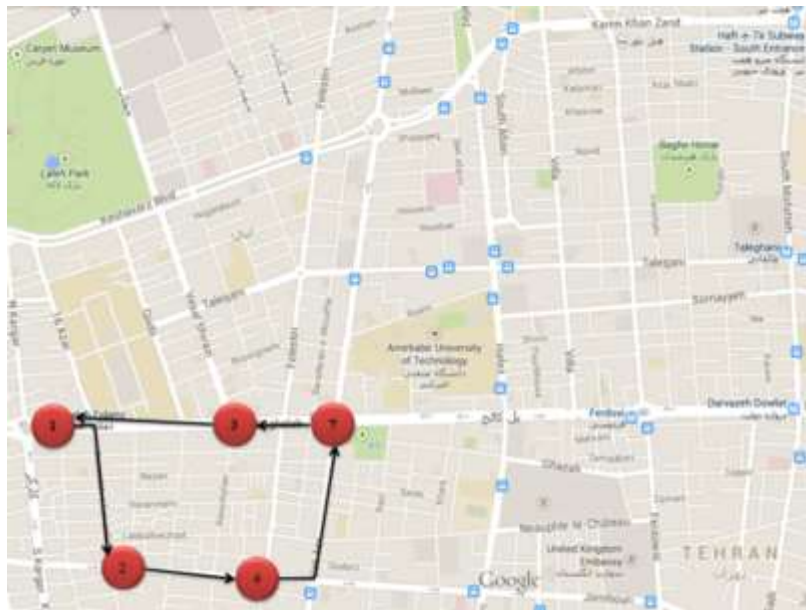


Figure 8. Optimally designed route of the second point of the Pareto front for Scenario #2

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Figure 9. Optimally designed route of the third point of the Pareto front for Scenario #3

8. Conclusion and future research

In this paper, a new application of covering-routing problem for the planning of public urban transportation network called Auxiliary Public Transportation Network (APTN) proposed to assist conventional public network during time periods of peak traffic. The objective of the proposed model was to design an APTN so that additional demand coverage maximized while the APTN costs minimized. The validation of the model was investigated considering one objective through sensitivity analysis of main parameters for an urban area in Tehran metropolis in Iran. Decision maker in charge of urban transportation company preferred to get a set of non-dominated solutions. Hence, the model is solved by using the ε -constraint method, based on three scenarios. According to an assumed secondary criterion, it concluded that the third point of the Pareto front for

Scenario #2 is the best. However, in order to determine the best solution, the secondary criteria should be got from the related company.

For future studies, development of the proposed model considering time window for each bus station is suggested. Also, multiple starts and finish points can be considered. Moreover, it is proper to incorporate a penalty function for the non-covered additional demands into the cost objective function. Finally, to actualize the APTN costs, it is proper to consider cash inflow and outflow in the second objective function.

REFERENCES

- [1] Angelelli, E., Speranza, M.G. (2002), *The Periodic Vehicle Routing Problem with Intermediate Facilities*. *European Journal of Operations Research*, 137, 233–247.;
- [2] Boffey, B., Narula, S.C. (1998), *Models for Multi-path Covering-routing Problems*. *Annals of Operations Research*, 82, 331–342;
- [3] Clarke, G., Wright, J.W. (1964), *Scheduling of Vehicles from a Central Depot to a Number of Delivery Points*. *Operations Research*, 12, 568–581;
- [4] Davari, S., Fazel Zarandi, M.H., Hemmati, A. (2011), *Maximal Covering Location Problem (MCLP) with Fuzzy Travel Times*. *Expert Systems with Applications*, 38, 14535–14541;
- [5] Doerner, K., Focke, A., Gutjahr, W.J. (2007), *Multi-criteria Tour Planning for Mobile Healthcare Facilities in a Developing Country*. *European Journal of Operations Research*, 179, 1078–1096;
- [6] Ehrgott, M., Xavier G. (2002), *Multi-objective Combinatorial Optimization—Theory, Methodology and Applications*. *Multiple criteria optimization: State of the art annotated bibliographic surveys*. Springer, US, 369–444;
- [7] Erbao, E., Mingyong, L. (2010), *The Open Vehicle Routing Problem with Fuzzy Demands*. *Expert systems with Application*, 37, 2405–2411;
- [8] Farahani, R.Z., Asgari, N., Heidari, N., Hosseini, M., Goh, M. (2012), *Covering Problems in Facility Location: A review*. *Computers and Industrial Engineering*, 62, 368–407;
- [9] Ghoseiri, K., Ghannadpour, S.F. (2010), *Multi-objective Vehicle Routing Problem with Time Windows Using Goal Programming and Genetic Algorithm*. *Applied Soft Computing*, 10, 1096–1107;
- [10] Golden, B.L., Magnanti, T.L., Nguyen, H.Q. (1977), *Implementing Vehicle Routing Algorithms*. *Networks*, 7, 113–148;

-
- [11] **Grandinetti, L., Guerriero, F., Laganá, D., Pisacane, O. (2012), *An Optimization-based Heuristic for the Multi-objective Undirected Capacitated Arc Routing Problem*. *Computers and Operations Research*, 39, 2300–2309;**
 - [12] **Hakimi, S.L. (1965), *Optimum Distribution of Switching Centers in a Communication Network and some Related Graph Theoretic Problems*. *Operations Research*, 13, 462–475;**
 - [13] **Jozefowiez, N., Semet, F., Talbi, E.G. (2007), *The Bi-objective Covering Tour Problem*. *Computers and Operations Research*, 34, 1929–1942;**
 - [14] **Jozefowiez, N., Semet, F., Talbi, E.G. (2008), *Multi-objective Vehicle Routing Problems*. *European Journal of Operations Research*, 189, 293–309;**
 - [15] **Keskin, B.B., Li, S., Steil, D., Spiller, S. (2012), *Analysis of an Integrated Maximum Covering and Patrol Routing Problem*. *Transportation Research Part E: Logistics and Transportation Review*, 48, 215–232;**
 - [16] **Khebbache-Hadji, S., Prins, C., Yalaoui, A., Reghioui, M. (2013), *Heuristics and Memetic Algorithm for the Two-dimensional Loading Capacitated Vehicle Routing Problem with Time Windows*. *Central European Journal of Operations Research*, 21, 307–336;**
 - [17] **Lamosa, M.J.P., Lamosa, D.M., De Marchi, M.M. (2011), *An Interactive Approach to Solve a Multi-objective Covering-routing Problem Considering Critical Areas*. *Cognitive Methods in Situation Awareness and Decision Support (CogSIMA)*; 2011 IEEE First International Multi-Disciplinary Conference on, 115–118;**
 - [18] **Meas, J.A., Boffey, T.B. (1996), *A Review of Extensive Facility Location in Networks*. *European Journal of Operations Research*, 95, 592–603;**
 - [19] **Mendoza, J.E., Castanier, B., Guéret, C., Medaglia, A.L., Velasco, N. (2010), *A Memetic Algorithm for the Multi-compartment Vehicle Routing Problem with Stochastic Demands*. *Computers and Operations Research*, 37, 1886–1898;**
 - [20] **Mirabi, M., FatemiGhomi, S.M.T., Jolai, F. (2010), *Efficient Stochastic Hybrid Heuristics for the Multi-depot Vehicle Routing Problem*. *Robotics and Computer-Integrated Manufacturing*, 26, 564–569;**
 - [21] **Nagy, G., Salhi, S. (2007), *Location-routing: Issues, Models and Methods*. *European Journal of Operational Research*, 177, 649–672;**
 - [22] **Park, J., Kim, B. (2010), *The School Bus Routing Problem: A Review*. *European Journal of Operational Research*, 202, 311–319;**

-
- [23] **Sáez, D., Cortés, C.E., Núñez, A. (2008), *Hybrid Adaptive Predictive Control for the Multi-vehicle Dynamic Pick-up and Delivery Problem Based on Genetic Algorithms and Fuzzy Clustering*. *Computers and Operations Research*, 35, 3412–3438;**
 - [24] **Tricoire, F., Graf, A., Gutjahr, W.J. (2012), *The Bi-objective Stochastic Covering Tour Problem*. *Computers and Operations Research*, 39, 1582–1592;**
 - [25] **Tricoire, F., Bostel, N., Dejax, P., Guez, P. (2013), *Exact and Hybrid Methods for the Multi-period Field Service Routing Problem*. *Central European Journal of Operations Research*, 21, 359–377;**
 - [26] **Yin, P., Mu, L. (2012), *Modular Capacitated Maximal Covering Location Problem for the Optimal Siting of Emergency Vehicles*. *Applied Geography*, 34, 247–254.**